12 COUNTERS
Answer Guide

2. Strategy analysis

   Player 1:
   The glaring weakness is placing a marker on 'square one', which is an impossible total to create by adding two dice. The notion of spreading out the discs is a good one, but students will find that squares 2 and 12 are the 'hardest', while squares such as 6, 7 or 8 are much 'easier'.

   Player 2:
   While it is correct that square 7 is the most likely, it only occurs with a probability of $\frac{6}{36} = \frac{1}{6}$.
   Hence it will take 6 rolls on average to release each disc and 6 x 12 = 72 rolls to remove all discs.

3. The table shows the most likely total is a 7, which can occur 6 different ways. The least likely totals are 2 and 12 which both occur in only one way.

Total	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

   For young students this table might be produced as a systematic list such as:
   7 ... (1 and 6), (2 and 5), (3 and 4), (4 and 3) etc
   6 ... (1 and 5), (2 and 4), etc
   One issue for students is whether (2, 4) is the same as (4, 2). One way to highlight they are different combinations is to use two different coloured dice.

4. With just one marker, the 'best' square to place it would be square 7, because this is the most likely result. The chance of throwing a total of 7 is 6 out of 36 or 1/6. This means on average it will take 6 rolls of the dice to remove this marker.

   With two markers, one of them should clearly be on the 7 - but where to put the other? If you put both on the 7, it will take, on average 6 rolls to remove each of them, ie: 12 rolls to remove both. If you put the second marker on the 6 (or the equally likely 8), the expected number of rolls to release them is easily shown empirically to be less than 12, hence this is a 'better' arrangement.

   With three markers, a class experiment to collect data between the possible best arrangements (suggested by students) would show which is 'best'. Students might suggest (6, 7, 8), ie: one on each of 6, 7 and 8, or (6, 7, 7).

5&6. The strategy to determine which is best raises the empirical versus theoretical debate. Initial observations such as the following can assist:

Option (ii) - putting all 12 on the 7 does not look good because you have to keep rolling 7s. This will take 12 lots of 6 = 72 rolls on average.

Clustering the markers on the most likely areas, but also spreading them out a bit seems to be the most effective.

One key strategy is to simplify the problem and find patterns which might assist in the more complex problem. This is the purpose of Question 4 above which would highlight the most likely squares being the centre numbers such as 6, 7 and 8.

The most likely strategy suggested might be to empirically test various combinations. Different groups in the class could run a set of trials on different combinations. This would quickly eliminate some unlikely combinations such as all 12 on the 7 or one in each square. A computer simulation, if available, might then be used to get the data needed to discriminate between combinations that seem 'close' to each other in likelihood.

A more sophisticated strategy might be:
Let's spread them out in proportion to the chances of that number occurring. That is put 1/6 of the discs on the 7, put 5/36 on the 6 and the 8, put 4/36 on the 5 and the 9 etc.

For 12 discs this gives 1/6 of 12 = 2 on the 7, 5/36 of 12 = 1.66 on the 6 and 8, etc., as the table shows:

2	3	4	5	6	7	8	9	10	11	12
0.33	0.5	1	1.33	1.66	2	1.66	1.33	1	0.5	0.33

This now requires a strategy for rounding off which might give a result such as 0,1,1,1,2,2,2,1,1,1,0. This could then be tested empirically against other possibilities.

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