Highest Number 1

Investigation Guide A: Discussion

2.　Analysing the game
i) the highest possible number in a single game is 654. This could happen in several ways but it is unlikely that if the player rolled a 4 first that they would put it in the ones column and also slightly unlikely that a 5 would be put in the tens column. Hence the most likely manner is for the dice to be rolled 6, 5 and 4 in that order.

ii) It follows that the greatest 5 game total would be 654 x 5 = 3270.

iii) The lowest possible result is 123, probably caused by rolls of 3, 2 and 1 in that order. The odds of this happening in just one game are 1/6 x 1/5 x 1/4 = once every 120 games.
[If 24 pupils in a class play 5 games each, this is 120 games which means it is reasonably likely that at least one will suffer the 'misfortune' of a 123 total.]
Consistent bad luck like this would produce a 5 game minimum of 123 x 5 = 615
[The chances of it happening 5 successive times to get a total score of 615 are extremely small [= $(1/120)^5$]

3.　Testing theories (also Q. 2, part iv)
a)　＿　3　＿　　　Second roll is a 4.
Pupils often say:
It doesn't matter where you put it because there will then be two bigger numbers (5 and 6) and 2 smaller numbers (1 and 2). So I will put it in the ones column and hope for a 5 or 6
(Implying they will not be 'punished' in the long run.)

However this is reasoning is not correct!
Placing the 4 in first position, ie:　4　3　＿, the possible outcomes (with equal likelihood) are 431, 432, 435 and 436 for an average expected score of 433.5
But with the 4 in the ones column, ie:　＿　3　4, the possible scores are 134, 234, 534 and 634 for an expected average of 384.

Hence the strategy of 4 in the hundreds column is better by 49.5 points per game.

A second way of reaching the same conclusion empirically is by setting up the situation and playing it 10 times both ways and averaging the scores.

Yet another view is to argue:
If I put the 4 in the hundreds column I'm assured of at least 400 points. But if I put it in the ones column and I throw a 1, I 'lose' about 300 points; if I throw a 2 I lose about 200 points; that's a total of about 500 points.

But if I throw a 5 or 6 I only 'gain' about 100 or 200 points, a total of only 300 points, so I have more to lose than to gain.

(Specifically a difference of 200 points spread over the 4 possibilities or about 50 per game.)

Other questions can be approached a similar way.

b) _ _ _2_ Second roll is a 4.
The 4 in the tens column produces possible results of 142, 342, 542 and 642 for an average of 417. The 4 in the hundreds column produces 412, 432, 452 and 462 for an average of 439.5 for a small advantage of 22.5 points for the 4 in the hundreds column.

c) _ _ _4_ Second roll is a 3.
The 3 in the tens column produces possible results of 134, 234, 534 and 634 for an average of 384. The 3 in the hundreds column produces results of 314, 324, 354 and 364 for an average of 344. Hence the tens position is slightly better by an average of 20 points per game.

d) _ _5_ _ Second roll is a 3.
The 3 in the hundreds position produces an average of 353.25, whereas the 3 in the ones column produces an average result of 378. Hence the 3 in the ones column is the best option by about 25 points per game.

e) _ _2_ _ Second roll is a 5.
The 5 in the hundreds position produces a result of 523.5 compared to 375 in the ones column. Hence it is a considerably better option (by 148 points per game).