Highest Number 1

Investigation Guide B: Discussion

1. Probability
The questions can be answered on several levels. Young students might answer descriptively. For example, in Qu. (i)
If I roll a 6 or a 5 I will win and there are only 4 cards left so I have a reasonable chance.
Another level of answer is to make a table of possibilities.
For example in Qu. (ii)

Opponent Rolls	Me
6	I Lose
5	I win with 3, 5 or 6
3 or 1	I win

It seems I have more chances to win than lose.

A more senior student may quantify the chances.
For example in Qu. (iii)
Pr (win) = Pr (my throw|opponent)
= Pr (6 | 2 or 3)
= 1/4 x 2/4
= 1/8 - so my chances are 12.5%.

1. i) Pr (W) = Pr (5 or 6) = 2/4 = 1/2 = 50 %
ii) Pr (W) = Pr (3 or 6 | 5) + Pr (anything | 1 or 3)
= 2/4 x 1/4 + 2/4 = 5/8 = 62.5%
iii) Pr (W) = Pr (6 | 2 or 3) = 1/4 x 2/4 = 1/8 = 12.5%
iv) Pr (W) = Pr (3 or 5 | 4) + Pr (anything | 1 or 3)
= 2/4 x 1/4 + 2/4 = 5/8 = 62.5%
v) Pr (W) = Pr (6 | 4) + Pr (anything | 1 or 2)
= 1/4 x 1/4 + 2/4 = 9/16 = 56.25%

2. The fund-raising story shell
This hypothetical context gives students a reason to scrutinise and interpret the results of their games and the results from other players. [Some schools have reported the class actually running the game at a school fete as a fund-raiser]
i) To neither win nor lose money 10 players @ $5 each would need to be above the cutoff score. From the sample results this score could be 2750.

iii) If 50 players each paid $1 to play, the income would be $50. From the results this table shows the expected payout for each total score.

Income = $50

Total score	3200	3100	3050	2500	2900	2725
Payout	nil	$5	$10	$145	$30	$65
Profit / Loss	+ $50	+ $45	+ $40	- $95	+ $20	- $15

For 300 players, if the above sample were to be repeated, the results would be 6 times the above figures, ie: + $300, + $270, + $240, - $570,

+ $120, - $90. [This raises the issue of sampling as a predictor of a whole population]

Clearly a score of 3200 'appears' to make the most profit, but is this really a good decision for the operators of the game? Pupils usually recognise that you need a few 'winners' to create interest - "if it is too hard, no one will play". Hence the decision of the 'best' score to be the prize winning target involves subjective judgement and the psychology of game playing.

The 50 data points in the table are only a sample - it is an assumption that another 50 would give the same results. How confident is the class that these results are a good predictor for 300 expected customers?
If we get it wrong, ie: set the prize too low, we could lose money, but if we set it too high no one will play!
The question of how big a sample you need to provide such confidence is an important mathematical question.

iv) There are various other combinations of entry and prize values. Some of these and the approximate break even score for each is:

$1 entry, $2 prize - break even score = 2,568
$1 entry, $10 prize - break even score = 2,750
$2 entry, $5 prize - break even score = 2,620
$2 entry, $15 prize - break even score = 2,900

3. Extensions
i) Removing the restriction of using each digit only once makes the highest possible score 666 and the lowest score 111. The strategy shift will not be great, since repeats are just as likely to be for low numbers as for high.

With no repeat digits, a strategy is composed of 30 possible combinations each requiring a decision. For example, for any one of the 6 first rolls, a decision is needed for any of the 5 possible second rolls. [No decision is needed for the third roll].

If repeats are allowed, then 36 decisions are needed to fully describe a strategy.

ii) 4 digit games
The maximum score is 6,543. The least is 1,234. Many strategies are possible. Each can be analysed using the logic described above in the discussion of Investigation Sheet Qu. 2.